tions are the same, and therefore the second ratios are equal too, i.e. $DE : AC = A'C' : AC$, which shows that $DE = A'C'$. We see now that in the right triangles DBE and $A'B'C'$, the hypotenuses and one of the legs are respectively congruent. Thus the triangles are congruent, and since one of them is similar to $\triangle ABC$, then the other one is also similar to it.

164. Theorem. ***In similar triangles, homologous sides are proportional to*** **homologous altitudes**, i.e. to those altitudes which are dropped to the homologous sides.

Indeed, if triangles ABC and $A'B'C'$ (Figure 171) are similar, then the right triangles BAD and $B'A'D'$ are also similar (since $\angle A = \angle A'$), and therefore

$$\frac{BD}{B'D'} = \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}.$$

EXERCISES

Prove theorems:

345. All equilateral triangles are similar.

346. All isosceles right triangles are similar.

347. Two isosceles triangles are similar if and only if their angles at the vertex are congruent.

348. In similar triangles, homologous sides are proportional to: (a) **homologous medians** (i.e. those medians which bisect homologous sides), and (b) **homologous bisectors** (i.e. the bisectors of respectively congruent angles).

349. Every segment parallel to the base of a triangle and connecting the other two sides is bisected by the median drawn from the vertex.

350. The line drawn through the midpoints of the bases of a trapezoid, passes through the intersection point of the other two sides, and through the intersection point of the diagonals.

351. A right triangle is divided by the altitude drawn to the hypotenuse into two triangles similar to it.

352. If a line divides a triangle into two similar triangles then these similar triangles are right.

353. Given three lines passing through the same point. If a point moves along one of the lines, then the ratio of the distances from this point to the other two lines remains fixed.

354. The line connecting the feet of two altitudes of any triangle cuts off a triangle similar to it. Derive from this that altitudes of any triangle are angle bisectors in another triangle, whose vertices are the feet of these altitudes.

355.* If a median of a triangle cuts off a triangle similar to it, then the ratio of the homologous sides of these triangles is irrational. Hint: Find this ratio.

Computation problems

356. In a trapezoid, the line parallel to the bases and passing through the intersection point of the diagonals is drawn. Compute the length of this line inside the trapezoid, if the bases are a units and b units long.

357. In a triangle ABC with sides a, b, and c units long, a line MN parallel to the side AC is drawn, cutting on the other two sides the segments $AM = BN$. Find the length of MN.

358. Into a right triangle with legs a and b units long, a square is inscribed in such a way that one of its angles is the right angle of the triangle, and the vertices of the square lie on the sides of the triangle. Find the perimeter of the square.

359. Two circles of radii R and r respectively are tangent externally at a point M. Compute the distance from M to the common external tangents of the circles.

3 Similarity of polygons

165. Definition. Two polygons with the same number of sides are called **similar**, if angles of one of them are respectively congruent to the angles of the other, and the homologous sides of these polygons are proportional. Thus, the polygon $ABCDE$ is similar to the polygon $A'B'C'D'E'$ (Figure 172), if

$$\angle A = \angle A', \ \angle B = \angle B', \ \angle C = \angle C', \ \angle D = \angle D', \ \angle E = \angle E',$$

and

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DE}{D'E'} = \frac{EA}{E'A'}.$$

Existence of such polygons is seen from the solution of the following problem.

166. Problem. *Given a polygon $ABCDE$, and a segment a, construct another polygon similar to the given one and such that its side homologous to the side AB is congruent to a* (Figure 173).